Team Round

1. Find the smallest positive integer N such that $N!$ is a multiple of 10^{2010}.

2. An equilateral triangle T is externally tangent to three mutually tangent unit circles, as shown in the diagram. Find the area of T.

3. The polynomial $p(x) = x^3 + ax^2 + bx + c$ has the property that the average of its roots, the product of its roots, and the sum of its coefficients are all equal. If $p(0) = 2$, find b.

4. A regular pentagon $P = A_1A_2A_3A_4A_5$ and a square $S = B_1B_2B_3B_4$ are both inscribed in the unit circle. For a given pentagon P and square S, let $f(P,S)$ be the minimum length of the minor arcs A_iB_j, for $1 \leq i \leq 5$ and $1 \leq j \leq 4$. Find the maximum of $f(P,S)$ over all pairs of shapes.

5. Let a, b, c be three three-digit perfect squares that together contain each nonzero digit exactly once. Find the value of $a + b + c$.

6. There is a big circle P of radius 2. Two smaller circles Q and R are drawn tangent to the big circle P and tangent to each other at the center of the big circle P. A fourth circle S is drawn externally tangent to the smaller circles Q and R and internally tangent to the big circle P. Finally, a tiny fifth circle T is drawn externally tangent to the 3 smaller circles Q, R, S. What is the radius of the tiny circle T?

7. Let $P(x) = (1+x)(1+x^2)(1+x^4)(1+x^8)(\cdots)$. This infinite product converges when $|x| < 1$. Find $P(\frac{1}{2010})$.

8. $P(x)$ is a polynomial of degree four with integer coefficients that satisfies $P(0) = 1$ and $P(\sqrt{2} + \sqrt{3}) = 0$. Find $P(5)$.

9. Find all positive integers $n \geq 3$ such that both roots of the equation

$$(n-2)x^2 + (2n^2 - 13n + 38)x + 12n - 12 = 0$$

are integers.

10. Let a, b, c, d, e, f be positive integers (not necessarily distinct) such that

$$a^4 + b^4 + c^4 + d^4 + e^4 = f^4.$$

Find the largest positive integer n such that n is guaranteed to divide at least one of a, b, c, d, e, f.