Tiebreaker Round

1. Let the series a_n be defined as $a_1 = 1$ and $a_n = \sum_{i=1}^{n-1} a_i a_{n-i}$ for all positive integers n. Evaluate $\sum_{i=1}^{\infty} \left(\frac{1}{4}\right)^i a_i$.

2. a, b, c, and d are distinct real numbers such that

$$a + \frac{1}{b} = b + \frac{1}{c} = c + \frac{1}{d} = d + \frac{1}{a} = x.$$

   Find $|x|$.

3. Find all ordered tuples (w, x, y, z) of complex numbers satisfying

$$\begin{aligned} x + y + z + xy + yz + zx + xyz &= -w \\ y + z + w + yz + zw + wy + yzw &= -x \\ z + w + x + zw + wx + xz + zwx &= -y \\ w + x + y + wx + xy + yw + wxy &= -z. \end{aligned}$$