

DUKE MATH MEET 2012

RELAY ROUND

1. Kathleen and Andrew play a game. Kathleen has a probability $0 < p < 1$ of winning the game. Andrew notes that Kathleen's probability of winning a best-out-of-3 series of games is the same as her probability of winning an individual game. What is the probability that Kathleen wins an individual game?
2. Andrew wants to figure out which pairs of his friends are also friends. He decides to survey every one of his F friends. It takes him 90 minutes to set up the survey, and then F minutes to give the survey to each of his friends. If $F = 60 \cdot TNYWR$, what is the time per friend that Andrew spends on the survey, including set-up?
3. Let $T = TNYWR/3$. $ABCD$ is a square; E is the midpoint of its diagonals, and F is a point external to the square such that $\angle CFD$ is right. If $FD = T$ and $FE = 12\sqrt{2}$, what is FC ?

4. How many ways can the numbers 1 through 6 be assigned to the vertices of a hexagon such that no two consecutive numbers lie on adjacent vertices? (Two numberings are considered equivalent if one can be rotated into the other.)

5. A palindrome is a string that reads the same way forwards and backwards - for example, “abba” and “xyzyx” are both palindromes. For $n \geq 1$, let $P(n)$ denote the number of palindromes of length n using the 26 letters of the English alphabet. Let $T = TNYWR$. Find $P(T+1)/P(T)$.

6. Let $T = TNYWR$. Let $f(x) = x^3 - 16x^2 + 10x - T$ have roots a, b, c . Find

$$\sum_{i=1}^2 \sum_{j=1}^2 \sum_{k=1}^2 a^i b^j c^k.$$