Duke Math Meet 2013-14
Individual Round

1. p, q, r are prime numbers such that $p^q + 1 = r$. Find $p + q + r$.

2. 2014 apples are distributed among a number of children such that each child gets a different number of apples. Every child gets at least one apple. What is the maximum possible number of children who receive apples?

3. Cathy has a jar containing jelly beans. At the beginning of each minute he takes jelly beans out of the jar. At the n-th minute, if n is odd, he takes out 5 jellies. If n is even he takes out n jellies. After the 46th minute there are only 4 jellies in the jar. How many jellies were in the jar in the beginning?

4. David is traveling to Budapest from Paris without a cellphone and he needs to use a public payphone. He only has two coins with him. There are three pay-phones - one that never works, one that works half of the time, and one that always works. The first phone that David tries does not work. Assuming that he does not use the same phone again, what is the probability that the second phone that he uses will work?

5. Let a, b, c, d be positive real numbers such that
$$a^2 + b^2 = 1;$$
$$c^2 + d^2 = 1;$$
$$ad - bc = \frac{1}{7}.$$
Find $ac + bd$.

6. Three circles C_A, C_B, C_C of radius 1 are centered at points A, B, C such that A lies on C_B and C_C, B lies on C_C and C_A, and C lies on C_A and C_B. Find the area of the region where C_A, C_B, and C_C all overlap.

7. Two distinct numbers a and b are randomly and uniformly chosen from the set $\{3, 8, 16, 18, 24\}$. What is the probability that there exist integers c and d such that $ac + bd = 6$?

8. Let S be the set of integers $1 \le N \le 2^{20}$ such that $N = 2^i + 2^j$ where i, j are distinct integers. What is the probability that a randomly chosen element of S will be divisible by 9?

9. Given a two-pan balance, what is the minimum number of weights you must have to weigh any object that weighs an integer number of kilograms not exceeding 100 kilograms?

10. Alex, Michael and Will write 2-digit perfect squares A, M, W on the board. They notice that the 6-digit number $10000A + 100M + W$ is also a perfect square. Given that $A < W$, find the square root of the 6-digit number.