Duke Math Meet 2017
Devil Round

January 7, 2018

1. Let $A = \{D, U, K, E\}$ and $B = \{M, A, T, H\}$. How many maps are there from A to B?

2. The product of two positive integers x and y is equal to 3 more than their sum. Find the sum of all possible x.

3. There is a bag with 1 red ball and 1 blue ball. Jung takes out a ball at random and replaces it with a red ball. Remy then draws a ball at random. Given that Remy drew a red ball, what is the probability that the ball Jung took was red?

4. Let $ABCDE$ be a regular pentagon and let AD intersect BE at P. Find $\angle APB$.

5. It is Justin and his $4 \times 4 \times 4$ cube again! Now he uses many colors to color all unit-cubes in a way such that two cubes on the same row or column must have different colors. What is the minimum number of colors that Justin needs in order to do so?

6. $f(x)$ is a polynomial of degree 3 where $f(1) = f(2) = f(3) = 4$ and $f(-1) = 52$. Determine $f(0)$.

7. Mike and Cassie are partners for the Duke Problem Solving Team and they decide to meet between 1pm and 2pm. The one who arrives first will wait for the other for 10 minutes, the lave. Assume they arrive at any time between 1pm and 2pm with uniform probability. Find the probability they meet.

8. The remainder of $2x^3 - 6x^2 + 3x + 5$ divided by $(x-2)^2$ has the form $ax + b$. Find ab.

9. Find m such that the decimal representation of $m!$ ends with exactly 99 zeros.

10. Let $1000 \leq n = \overline{\text{DUKE}} \leq 9999$. be a positive integer whose digits $\overline{\text{DUKE}}$ satisfy the divisibility condition:
$$1111 \mid \left(\overline{\text{DUKE}} + \overline{\text{DU}} \times \overline{\text{KE}}\right)$$
Determine the smallest possible value of n.