Duke Math Meet
Team Round

January 7, 2018

1. What is the maximum possible value of m such that there exist m integers $a_1, a_2, ..., a_m$ where all the decimal representations of $a_1!, a_2!, ..., a_m!$ end with the same amount of zeros?

2. Let $f : \mathbb{R} \to \mathbb{R}$ be a function such that
$$f(x) + f(y^2) = f(x^2 + y), \text{ for all } x, y \in \mathbb{R}.$$
Find the sum of all possible $f(-2017)$.

3. What is the sum of prime factors of 1000027?

4. Let
$$\frac{1}{2!} + \frac{2}{3!} + ... + \frac{2016}{2017!} = \frac{n}{m},$$
where n, m are relatively prime. Find $(m - n)$.

5. Determine the number of ordered pairs of real numbers (x, y) such that
$$\sqrt[3]{3 - x^3 - y^3} = \sqrt{2 - x^2 - y^2}.$$

6. Triangle $\triangle ABC$ has $\angle B = 120^\circ$, $AB = 1$. Find the largest real number x such that $CA - CB > x$ for all possible triangles $\triangle ABC$.

7. Jung and Remy are playing a game with an unfair coin. The coin has a probability of p where its outcome is heads. Each round, Jung and Remy take turns to flip the coin, starting with Jung in round 1. Whoever gets heads first wins the game. Given that Jung has the probability of $\frac{8}{15}$, what is the value of p?

8. Consider a circle with 7 equally spaced points marked on it. Each point is 1 unit distance away from its neighbors and labelled 0,1,2,...,6 in that order counterclockwise. Feng is to jump around the circle, starting at the point 0 and making six jumps counterclockwise with distinct lengths $a_1, a_2, ..., a_6$ in a way such that he will land on all other six nonzero points afterwards. Let s denote the maximum value of a_i. What is the minimum possible value of s?

9. Justin has a $4 \times 4 \times 4$ colorless cube that is made of 64 unit-cubes. He then colors m unit-cubes such that none of them belong to the same column or row of the original cube. What is the largest possible value of m?

10. Yikai wants to know Liang's secret code which is a 6-digit integer x. Furthermore, let $d(n)$ denote the digital sum of a positive integer n. For instance, $d(14) = 5$ and $d(3) = 3$. It is given that

$$x + d(x) + d(d(x)) + d(d(d(x))) = 999868.$$

Please find x