Relay Round

DMM 2022

Problem 1.1: A robot is located at 2 on the number line, and it needs to reach either 5 or 0. Every second, there's a $\frac{1}{3}$ chance it breaks down, a $\frac{1}{3}$ chance it moves one unit in the positive direction, and a $\frac{1}{3}$ chance it moves one unit in the negative direction. The probability the robot manages to reach 5 or 0 before breaking down is $\frac{m}{n}$, where m and n are coprime. Find n.

Your Answer:

Problem 1.2: Let T = TNYWR. Navya, the fruit ninja, has a bitter feud with watermelon and strawberries. She can only cut 3 watermelon with one slice or T strawberries with one slice. Suppose she slices 17 times tomorrow, and let N be the total number of watermelon and strawberries she cuts tomorrow. How many possible values of N are prime?

Your Answer:

Problem 1.3: Let $T = \text{TNYWR}$ and $f(x) = x^5 + 18x^4 + 19x^3 + 20x^2 + 21x + T$. The roots of f are a, b, c, d and e. Find $(a-1)(b-1)(c-1)(d-1)(e-1)$.

Your Answer:

Problem 2.1: $x, y \in \mathbb{R}$ satisfies $x\sqrt{y-1} + y\sqrt{x-1} = xy$. Find x.

Your Answer:

Problem 2.2: Let $T = \text{TNYWR}$. A sequence $\{a_n\}$ satisfies that for any $m, n \in \mathbb{N}$ such that $m \geq n$ we have $a_{m+n} + a_{m-n} = \frac{1}{T}(a_{2m} + a_{2n})$. Given $a_1 = 1$, find the last digit of a_{2023}.

Your Answer:

Problem 2.3: Let $T = \text{TNYWR}$. The sequence $\{a_n\}$ satisfies $a_1 = 7$ and the recurrence relation

$$a_{n+1} = Ta_n + 7.$$

Find the sum of all values of i such that a_i is a divisor of a_{88}.

Your Answer: