2023 DUKE MATH MEET RELAY ROUND

1 Relay Round - Set 1

1. Michelle has 10 of her students standing in a row. In how many ways can she pick a nonempty subset of students for a class activity such that for each pair of adjacent students in the row, at most one of them is picked?
2. Let $T = \text{TNYWR}$. Find the smallest integer n such that $n \geq T$ and $17(17^n - 3^n - 7)$ is divisible by 2023?
3. Let T be the smallest perfect square that is greater than or equal to TNYWR. Find the positive integer i satisfying
$$i \leq \sum_{k=1}^{T} \frac{1}{\sqrt{k}} < i + 1$$

2 Relay Round - Set 2

1. The chess players Akshar and Rishabh are playing a series that can last up to 15 games. A game either results in a win (1 point to the winner, 0 to the loser) or a draw (half a point to each). The series ends when the player with fewer total points can no longer theoretically catch up to the player with more total points, or when 15 games are complete. How many pairs of total points (x, y), where x is Akshar's total points and y is Rishabh's total points, are possible once the series ends?
2. Let T be the quotient when TNYWR is divided by 3. Kevin's favorite polynomial is
$$P(x) = x^4 - Tx^3 + Tx - 1.$$
Let $a < b < c < d$ be the roots of $P(x)$. Find the value of
$$b^a + b^c + d^a + d^c.$$
3. Let $T = \text{TNYWR}$. In 2-dimensional space, the area enclosed by the graph $\frac{x^2+y^2}{|x-y|} < T$ can be expressed as $a\pi$. Find a.