Tiebreaker Round

DMM 2023

1 Tiebreaker

1. Consider the polynomial
$$p(x) = x^3 + x^2 - 1.$$
Let r_1, r_2, and r_3 be the roots of of $p(x)$. Find the value of
$$r_1^4 + r_2^4 + r_3^4.$$

2. Calvin has 10 courses he wishes to take, each numbered 1,2,...,10. However, course 1 serves as a prerequisite to course 2, course 3 is a prerequisite for course 4, etc.. In how many different orders can Calvin take the courses given that he can only take one course at a time and that the prerequisite conditions must be satisfied for all courses (i.e. course 1 must be taken before course 2)?

   Please answer with a specific number, not a formula.

3. You have 4 6-sided dice. One die has $\sin^2(e)$ on half of the faces and 0 on the other half. The other 3 dice are identical, each has $\cos^2(e)$ on one face and 0 on every other face. You roll each die once, what is the expected value of the sum of all the dice?