Guts Round - Set 1

1. [**1**] How many ways can you arrange the letters $DUKEUNIVERSITY$ so that the two U's are not next to each other? The answer can be expressed as $\frac{a!}{8} - \frac{b!}{4}$. Find $a + b$.
2. [**1**] Compute $(\log 2000)^{\log\log 200} - (\log 200)^{\log\log 2000}$ where all logarithms are base 10.
3. [**1**] Define sequence a_n such that $a_n = na_{n-1}$ and $a_1 = 1$. What is the largest power of 3 that divides a_{2024}?

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Guts Round - Set 2

1. [**2**] Let $f(n)$ be a recursive function defined as follows:

$$f(n) = \begin{cases} n, & \text{if } n \leq 2 \\ f(n-1) + 2 \cdot f(n-2), & \text{if } n > 2 \end{cases}$$

   Compute $\log_2(f(2024))$

2. [**2**] Andrew and Brandon roll two standard dice. Andrew multiples the two numbers rolled, while Brandon adds the two numbers. Given that the probability that Andrew's product is greater than Brandon's sum can be written in simplest form as $\frac{m}{n}$, find $m + n$.
3. [**2**] Let S_1 be a square. Let C_1 be the circle inscribed in the S_1. For all $n \in \mathbb{N}$ with $n > 1$, let S_n be the square inscribed inside of C_{n-1} and let C_n be the circle inscribed inside of S_n. Let each region contained in S_n but not C_n be shaded. If a dart is thrown at S_1 uniformly at random, the probability that it lands in the shaded region can be expressed as $a - \frac{b\pi}{c}$ with $a, b, c \in \mathbb{Z}$. What is $a + b + c$?

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Guts Round - Set 3

1. [**3**] Let (k, n) be a solution to the equation: $2024^k - 3n = 1$ such that $\frac{n+1}{k+1}$ is an integer. Find the largest possible value of $\frac{k+n}{3}$.

2. [**3**] Let a, b, and c be the solutions to the equation $x^3 - 11x^2 + 56x - 6$. Compute the sum of all possible values of:

$$\left(1 + \frac{a^2}{b^2}\right)\left(1 + \frac{b^2}{c^2}\right)\left(1 + \frac{c^2}{a^2}\right)$$

3. [**3**] What is the area of the shape in the complex plane formed by connecting all points x satisfying $x^3 - (i\sqrt{3} + 1)x^2 + (i\sqrt{3} - 1)x + 1 = 0$ and y satisfying $y^4 = 1$? The answer can be expressed $\frac{a+\sqrt{b}}{c}$, where b is square-free. Find $a + b + c$.

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Guts Round - Set 4

1. [**4**] Quadrilateral $ABCD$ is inscribed in a circle such that $AB = BC = 6$, $AD = 4$, and $\angle CDA = 120°$. The length BD can be expressed as $a\sqrt{b} + c$. Find $a + b + c$

2. [**4**] Suppose a, b, c, d are the zeroes of $x^4 - 3x^3 - 33x^2 + 79x - 36$. The absolute value of the cyclic sum:

$$\sum_{\text{cyc}} \frac{1}{abc - (d-1)^2 + 36}$$

can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m+n$. We define the cyclic sum to be the sum of the terms across all 4 cyclic permutations of assigning the roots to a, b, c, d (i.e. $(a, b, c, d), (b, c, d, a), (c, d, a, b), (d, a, b, c)$).

3. [**4**] A container holds a certain number of identical balls. The ratio of the total volume of the balls to the volume of the empty space in the container is $1 : k$, where k is an integer greater than 1. After removing a prime number of balls, the ratio of the total volume of the remaining balls to the empty space becomes $1 : k^2$. Find the initial number of balls in the container.

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Guts Round - Set 5

1. [**5**] Find the sum of all possible integer values of n such that $\frac{9n+10}{4n+17}$ is the square of a rational number.

2. [**5**] Compute the number of ordered triples of positive integers (a, b, c) that satisfy

$$\left(1+\frac{1}{a}\right)\left(1+\frac{1}{b}\right)\left(1+\frac{1}{c}\right)=2$$

3. [**5**] Let

$$f(n)=\begin{cases} x/2, & \text{if } n \text{ even} \\ x+1, & \text{if } n \text{ odd} \end{cases}$$

   Let $g(x)$ be the minimum k such that $f^k(x) = f(f(...f(x))) = 1$. Note that $g(1) = 0$ by convention). Compute $\sum_{x=1}^{64} g(x)$.

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Guts Round - Set 6 (Estimation)

1. Annually, the amount of energy that is used to power Duke could be used to power how many homes?

2. How much weight, in pounds, do all the books in Duke's Libraries weight?

3. How many characters of LaTeX are contained in the Overleaf document with this year's DMM problems?

PROBLEM 1: _________ **PROBLEM 2:** _________ **PROBLEM 3:** _________

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