2024 DUKE MATH MEET RELAY ROUND

1 Relay Round - Set 1

1. Given regular pentagon $ABCDE$, point F is inside the pentagon such that ABF is an equilateral triangle. Find the measure of $\angle FCD$ in degrees.
2. Let $T = \text{TNYWR}$. How many ways are there to arrange the digits of T and one more digit from the set $\{0, 1, 2, 3, 4, 5\}$ so that the resulting number is divisible by 4 and greater than 100?
3. Let $T = \text{TNYWR}$. Let $AEDF$ and $FDGH$ be squares that share a common side, and let B be on line AE so that E lies in between points A and B. Extend line BD to intersect side HF at C. Given that $AC = CG = T/6$, compute the area of triangle ABC.

2 Relay Round - Set 2

1. How many positive integers less than 64 have at most two 1s in their binary representation?
2. Let $T = \text{TNYWR}$. Suppose that $x^2 + \frac{1}{x^2} = \frac{T}{3}$ for some real value x. Compute the product of all possible values of $x^3 + \frac{1}{x^3}$.
3. Let $T = \text{TNYWR}$. Let K be the last digit of T. Two players each roll two 6-sided dice, first player A, then player B. If player A rolls a sum of K, they win. If player B rolls a sum of $K + 1$, they win. They take turns, back and forth, until someone wins. Find the probability that A wins, which can be expressed as a simplified fraction $\frac{a}{b}$.