Tiebreaker Round

DMM 2024

Tiebreaker 1

Let $f_1(x) = x + 3$, and for $n \geq 1$ define

$$f_{n+1}(x) = (1 + x)f_n(x) + (-1)^n(2x + 4).$$

Then the two real roots of $f_{2024}(x)$ can be expressed as $a \pm b\sqrt[2024]{c}$, for some integers a, b, c with b and c positive. What is $a + b + c$?

Tiebreaker 2

A 11×11 square is divided into 121 unit squares, forming a 12×12 grid of vertices. How many ways can we color these vertices either red or blue such that each unit square has exactly 2 red vertices?

Tiebreaker 3

Suppose you have a quadratic Q, defined by $y = \frac{x^2}{22} + 22$. A circle C intersects Q at four points, the x-coordinates of three of these points being 7, -6, and -23. Find the coordinates of the last intersection point.