Guts Round - Set 1

1. [**1**] Consider the sequence $2, 0, 2, 5, 2, 0, 2, 5, 2, \dots,$ where the four numbers $2, 0, 2, 5$ repeat indefinitely. Let b_n be the sum of the first n terms of this sequence. Find the unique positive integer k such that $b_k = 2025$.

2. [**1**] Find the number of ordered triples of nonnegative integers (a, b, c) satisfying

$$a + \frac{b}{2} + \frac{c}{4} = 20.25$$

3. [**1**] The number 2025 has the special property that the fourth digit divides the second digit (5 divides 0) and the third digit divides the first digit (2 divides 2). How many four digit numbers greater than 1000 have this property? (**Note:** 0 does not divide 0)

PROBLEM 1: _________ **PROBLEM 2:** _________ **PROBLEM 3:** _________

TEAM NAME: ___________________________

Guts Round - Set 2

1. [**2**] The Duke Blue Devil's favorite numbers are of the form $\overline{DUKE}$, where D is non-zero and $\overline{DUKE}$ has the same number of factors as 2025. How many favorite numbers does he have?

2. [**2**] In rectangle $DUKE$, diagonal $\overline{UE}$ is drawn along with altitudes $\overline{DX}$ of $\triangle EDU$ and $\overline{KY}$ of $\triangle UKE$. Given that $UK = 20$, and $UE = 25$, compute XY.

3. [**2**] How many four digit numbers can be expressed as $\overline{DUKE}$, where D, U, K, E are not necessarily distinct integers and $0 < D, U, K, E < 10$ such that every permutation of its digits is divisible by 4 ($\overline{DUKE}$, $\overline{DEUK}$, etc).

PROBLEM 1: _________ **PROBLEM 2:** _________ **PROBLEM 3:** _________

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Guts Round - Set 3

1. [**3**] Consider the tetrahedron $T = \{(x, y, z) \in \mathbb{R}^3 \mid x, y, z \geq 0, x + y + z \leq 1\}$. How many points of the form $(\frac{a}{6}, \frac{b}{6}, \frac{c}{6})$, with a, b, and c integers, lie on the surface of the tetrahedron?

2. [**3**] In triangle ABC, we have that $AB = 10$, $AC = 9$, and $BC = 17$. Let point P lie on $\overline{AB}$ such that $AP = 4$ and point Q lie on $\overline{AC}$ such that $AQ = 3$. Let T be the intersection of $\overline{BQ}$ and $\overline{CP}$. Let D be the intersection of line AT and the line through B parallel to $\overline{AC}$. Find the area of quadrilateral $ABDC$.

3. [**3**] Alice and Bob are playing a game in which, on each turn, they must name a prime number. On the first turn, Alice names the number 2. On each turn, a player can add a positive integer to the current number to get another prime number, but they cannot add more than two times what is necessary to do so. In other words, if the current prime number is n, and the smallest prime number greater than n is p, the player can only add at most $2(p - n)$ to n to yield another prime number.

   Define game G_q, for positive prime q, such that the game ends when a player names q, and that player is declared the winner. Assuming optimal play from Alice and Bob, find the sum of all q such that Bob has a winning strategy in the game G_q.

PROBLEM 1: _________ **PROBLEM 2:** _________ **PROBLEM 3:** _________

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Guts Round - Set 4

1. [**4**] You glue together equilateral, equal size triangles edge-to-edge into a hollow solid such that each edge is shared by exactly two triangles, and there are no holes/punctures. Suppose that around each vertex, either 5 triangles meet or 6 triangles meet, and there are no other possibilities. How many vertices have 5 triangles meeting around them?

2. [**4**] Square $ABCD$ with side length 4 has points E and F on sides $\overline{CD}$ and $\overline{AB}$, respectively, such that E is the midpoint of $\overline{CD}$ and $AF = 3$. Let G be a point on $\overline{EF}$ and H on $\overline{BC}$ such that $\triangle DEF \sim \triangle GHC$. If $\overline{EG}$ can be expressed as $\frac{a\sqrt{b}}{c}$, where b is not divisible by the square of any prime. Find $a + b + c$.

3. [**4**] You have a Christmas tree with 3 layers (top, middle, and bottom) and want to decorate it with 50 indistinguishable ornaments. How many ways can you distribute them across the three layers such that each layer has at least one ornament, and each layer has strictly more ornaments than the layer above it (i.e., the bottom layer has more ornaments than the middle layer, which has more ornaments than the top layer)?

PROBLEM 1: ________ **PROBLEM 2:** ________ **PROBLEM 3:** ________

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Guts Round - Set 5

1. [**5**] You have a ranked choice voting system with 17 candidates and k voters, where each voter has one ballot where they rank their 1st, 2nd, ... 16th, and 17th choices. Call a collection of ballots **super-non-condorcet** if there exists a permutation of all the candidates S1, S2... S17 such that S1 beats S2, S2 beats S3, ... S16 beats S17, S17 beats S1 (where A beats B if strictly more voters ranked A above B). Find the minimum value of k where there exists a **super-non-condorcet** collection of k ballots.

2. [**5**] Let x_n, y_n, for $n \geq 0$, be infinite sequences of positive numbers such that $x_n = c \cdot y_n$ for some constant c, and $x_n^2 + (y_0 + y_1 + ... + y_n)^2 = 2500$ for all n. If $x_0 = 50 - 2^{-50}$, find

$$\left\lfloor \sum_{i=0}^{\infty} x_i \cdot y_i \right\rfloor$$

3. [**5**] A permutation p of length n is an arrangement of the integers $1, 2, 3, \ldots, n$. For example, $(3, 1, 2)$ and $(1, 3, 2)$ are permutations of length 3. If the ith number in the permutation is i, then we call i a fixed point. For example, in $(1, 3, 2)$, 1 is the only fixed point. How many permutations $(p_1, p_2, \ldots, p_{12})$ of length 12 are there with three fixed points, such that $p_1 = 3, p_2 = 6, p_3 = 4$?

PROBLEM 1: _________ **PROBLEM 2:** _________ **PROBLEM 3:** _________

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Guts Round - Set 6 (Estimation)

1. Akshar wants to build Pokémon gyms at the 3 northernmost state capitals in the continental US. What is the area in mi^2 of the triangle formed by these three Pokémon gyms?

2. Jaemin has just discovered that Spiritombs are real! Given that every Homo sapiens in history became a Spiritomb when they passed away and that Spiritombs are uniformly distributed on the surface of the Earth, how many Spiritombs live in Jaemin's lawn (area: 1 mi^2)?

3. Nikhil recently found out that Pikachu releases the same electrical output as 100 adult electric eels. How many Pikachus would you need to power the average U.S. home for 24 hours?

PROBLEM 1: _________ **PROBLEM 2:** _________ **PROBLEM 3:** _________

TEAM NAME: ___________________________