2025 DUKE MATH MEET INDIVIDUAL ROUND

Problems 1-2 **Name** ________________

Time Limit: 10 minutes **Team** ________________

Problem 1

Thomas wants to go Go-Karting, but forgot the name of the Go-Kart place. He knows that it consists of two letters of the 26-letter English alphabet (any letter from A-Z). He also knows that the second letter does not appear earlier in the alphabet than the first letter. For example, DU, and MU are possible names, but NC is not. How many possible names are there for the Go-Kart place?

Problem 2

How many integers from 1 to 2025, inclusive, can be expressed in the form $\lfloor 2x \rfloor + \lfloor 0x \rfloor + \lfloor 2x \rfloor + \lfloor 5x \rfloor$ for some positive real number x?

ANSWER TO PROBLEM 1

ANSWER TO PROBLEM 2

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Problems 3-4 **Name** ____________________

Time Limit: 10 minutes **Team** ____________________

Problem 3

Consider triangle ABC. Let $v(A), v(B), v(C)$ be (possibly negative) real number values assigned to the vertices of A, B, C. The value of an edge is the sum of the values of its endpoints, and the capacity of an edge AB, denoted $c(AB)$, is the maximum allowed value of edge AB. If $c(AB) = 11, c(BC) = 8, c(AC) = 25$, what is the largest possible value of $v(A) + v(B) + v(C)$?

Problem 4

Fix set $S = \{-3, -2, -1, 0, 1, 2, 3\}$. Let $P(x, y)$ be a real polynomial in two variables with both the degree of x and the degree of y equal to 6. Assume that $P(a, b) = 0$ for all pairs (a, b) such that $a \in S$ and $b \in S$, except possibly $(0, 0)$, and suppose the coefficient of x^6y^6 is 1. Compute $P(0, 0)$.

ANSWER TO PROBLEM 3

ANSWER TO PROBLEM 4

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Problems 5-6 **Name** ________________

Time Limit: 10 minutes **Team** ________________

Problem 5

Let $A = (0, 0)$ and $B = (20, 0)$. Define ω to be the circle consisting of all points X satisfying

$$\frac{XA}{XB} = \frac{3}{2}.$$

Let γ be the circle with diameter $\overline{AB}$. Circles ω and γ intersect at distinct points M and N. The length MN can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

Problem 6

Let p, q be primes and r a positive integer such that

$$1 + p = q^r$$

For a fixed value p, we can express $p - 1$ as the product of primes as follows

$$p - 1 = 2 \times 3 \times 107 \times 6361 \times 69431 \times 20394401 \times 28059810762433$$

for which there exists a unique solution (q, r). Find $q + r$.

ANSWER TO PROBLEM 5

ANSWER TO PROBLEM 6

2025 DUKE MATH MEET INDIVIDUAL ROUND

Problems 7-8 **Name** ________________

Time Limit: 10 minutes **Team** ________________

Problem 7

Call a positive integer *sendy* if it consists of only the digits 6 and 7. Leo plays the following game with this number. He can choose to either delete the first digit (given it has at least one digit) and score 1 point, or delete the first two digits if they are exactly 67 (given it has at least two digits), and score 3 points. The game ends when the number has no more digits left. The **score** of a *sendy* number is the maximum number of points Leo can achieve while playing this game.

For example, the **score** of 6767 is 6 points because Leo can delete 67 twice, for 3 points each. What's the largest possible **score** for a *sendy* number that has 500 digits and is divisible by 11?

Problem 8

You have 8 indistinguishable black chocolate wafer discs and 4 indistinguishable white cream filling discs. When you arrange these discs in a random order (with each possible arrangement equally likely), you can sometimes form a "complete oreo" - defined as a non-overlapping pattern "BWB" when reading from left to right.

For example:

- The pattern "BWB" counts as one complete oreo.
- BWBWB counts as one complete oreo (not two, as the patterns overlap), and so does BWBBWW.
- We want to count the maximum number of oreos: BWBWBWB counts as two complete oreos and not one.

If all 12 pieces are arranged in a random order, then the expected number of complete oreos that will appear can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

ANSWER TO PROBLEM 7

ANSWER TO PROBLEM 8

2025 DUKE MATH MEET INDIVIDUAL ROUND

Problems 9-10 Name ________________

Time Limit: 10 minutes Team ________________

Problem 9

The cubic polynomial $x^3 - 20x^2 + bx - 125 = 0$ has three real roots that are consecutive terms in a geometric sequence. Find b.

Problem 10

In trapezoid $ABCD$, $\angle A = \angle D = 90^\circ$. Let M and N be the midpoints of diagonals $\overline{AC}$ and $\overline{BD}$, respectively. Let Q and R be the other intersections of $\overline{BC}$ and the circles that go through points A, B, N and C, D, M respectively. Denote P as the midpoint of $\overline{QR}$, L as the midpoint of $\overline{BC}$, and T as the foot of the altitude from B to $\overline{CD}$. Let K be the midpoint of $\overline{MN}$. If $KL = 4$ and $PL = 2$, $\frac{TC}{BC}$ can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

ANSWER TO PROBLEM 9

ANSWER TO PROBLEM 10