2025 DUKE MATH MEET TIEBREAKER ROUND

Tiebreaker 1

Let $f(2x) = 2f(x)g(x)$ and $g(2x) = (g(x))^2 - (f(x))^2$. If $f(1) = 1$ and $g(1) = \sqrt{3}$, $f(1024)$ can be expressed as $a^b\sqrt{c}$, where a and c are positive integers that are not divisible by the square of any prime. Find $a + b + c$.

Tiebreaker 2

Consider four-digit number $\overline{ABCD}$. This number is special if $D^2 - A^2 - B^2 - C^2$ is a positive perfect square. What is the largest special four-digit number?

Tiebreaker 3

In how many ways can you arrange the letters of $DUKEUNIV$ so that no two adjacent letters are the same?